Let A be the event "the coin is genuine", B be the event "the device is working properly" and F_k be the event "the device flashes green k times in a row". Our priors are $p(A) = 0.5$ and $p(B) = 0.99$.
We want to compute our level of confidence that the coin is genuine when we observe k green flashes, which corresponds to the conditional probability $p(A|F_k)$.

$$p(A|F_k) = \frac{p(A \cap F_k)}{p(F_k)}.$$

By law of total probability we may write

$$\begin{aligned} p(A \cap F_k) &= p(A \cap F_k \cap B) + p(A \cap F_k \cap \bar{B}) \\ &= p(F_k|A \cap B)p(A \cap B) + p(F_k|A \cap \bar{B})p(A \cap \bar{B}). \end{aligned}$$

The events A and B are independent so that

$$\begin{aligned} p(A \cap B) &= \frac{1}{2} \cdot 0.99, \\ p(A \cap \bar{B}) &= \frac{1}{2} \cdot 0.01. \end{aligned}$$

The event $F_k|A \cap B$ is "observing k green flashes with a genuine coin and a working device", so

$$p(F_k|A \cap B) = 0.9^k.$$

The event $F_k|A \cap \bar{B}$ is "observing k green flashes with a genuine coin and a faulty device", which has probability 1. In total:

$$p(A \cap F_k) = 0.9^k \cdot \frac{1}{2} \cdot 0.99 + \frac{1}{2} \cdot 0.01.$$

Looking at the denominator now, two applications of the law of total probability yields

$$p(F_k) = p(F_k \cap A \cap B) + p(F_k \cap \bar{A} \cap B) + p(F_k \cap A \cap \bar{B}) + p(F_k \cap \bar{A} \cap \bar{B}).$$

The first and third terms were computed previously. The second term is

$$p(F_k \cap \bar{A} \cap B) = p(F_k|\bar{A} \cap B)p(\bar{A} \cap B),$$

so it corresponds to the setting where the coin is fake, but the device is working and flashes green k times, hence

$$p(F_k \cap \bar{A} \cap B) = \frac{1}{2} \cdot 0.99 \cdot (0.1)^k.$$

The fourth term is

$$p(F_k \cap \bar{A} \cap B) = p(F_k|\bar{A} \cap \bar{B})p(\bar{A} \cap \bar{B}),$$

so it corresponds to the setting where the coin is fake, the device is faulty and flashes green k times. The term on the left is just 1 since the device will always flash green in this scenario, so

$$\begin{aligned} p(F_k \cap \bar{A} \cap B) &= p(\bar{A} \cap \bar{B}) \\ &= \frac{1}{2} \cdot 0.01. \end{aligned}$$

Putting everything together,

$$p(F_k) = 0.01 + \frac{1}{2} \cdot 0.99 \cdot \left(0.9^k + 0.1^k\right),$$

and so replacing in the very first equation,

$$p(A|F_k) = \frac{0.9^k \cdot \frac{1}{2} \cdot 0.99 + \frac{1}{2} \cdot 0.01}{0.01 + \frac{1}{2} \cdot 0.99 \cdot (0.9^k + 0.1^k)}.$$

Computationally we find a maximum for $k = 3$ with confidence 0.985 and a confidence of 0.54 for $k = 60$.